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RECURSIVE ESTIMATION WITH NON-HOMOGENEOUS IMAGE MODELS.(U)
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RECURSIVE ESTIMATION WITH NON-HOMOGENEOUS IMAGE MODELS*

Vinay K. Ingle, Assad Radpour, John W. Woods, Howard Kaufman

Electrical and Systems Engineering Department
Rensselaer Polytechnic Institute
Troy, New York 12181

Summary

This paper presents initial results on spatially variant recursive estimation of images. Parameters of block-wise constant recursive model are identified on noise free data. The models are then used to design reduced update Kalman filters which are applied to noisy data. The results are presented and discussed.

Introduction

Digital processing of images has in recent years become both economic and practical. Most sophisticated image processing is performed off line on large machines because of the large memory and computational requirements of the presently used non-recursive methods. In particular for the image estimation problem, classical nonrecursive techniques involve operations with large matrices and their inverses and are hence not suitable for real-time applications which might include:

1) Restoration of noisy images after reception on a low power transmission link.
2) Pictures arising from low light level imaging where back sensor noise significantly contributes to the output signal.
3) Reception of a decoded DPCM image which results from a maximum-likelihood decoding technique.
4) Processing of non-image two-dimensional (2-D) data for noise reduction prior to display in image format.

Previous efforts towards the development of recursive 2-D filters have resulted in algorithms which for the most part take advantage of one-dimensional approximations or require a state vector with an exceptionaly large number of components (>100). Furthermore these algorithms do not take into account the non-stationarity of the image being processed. Thus a constant coefficient two-dimensional model has in general been apriori assumed for the entire picture.

Consequently, in view of the need for developing a two-dimensional recursive filter suitable for processing non-stationary images, we have combined a least squares parameter identification procedure with the previously developed 2-D reduced update Kalman filter.[1]

More precisely we consider image models as given by a half-plane Markovian model[2], which is observed in the presence of an additive white Gaussian noise field. Given a set of measurements and values for the model coefficients, a Kalman type estimator can be designed for estimating the image pixel intensities.[1] Computation can be greatly reduced by limiting the update process at point (m,n) to only those elements of the image which are directly coupled to point (m,n) via the Markovian model. If however, the image is non-stationary then the coefficients will vary as a function of the coordinates (m,n). In this case it is desirable that on-line identification of the coefficients be performed in conjunction with the state estimation.

To this effect, least squares identification was used to find estimates of the coefficients over blocks of the image. These estimates were then used by the reduced update Kalman filter for processing those image points contained within the pertinent block. Results indicate that this procedure significantly improves on the results obtained using a spatially invariant model.

Reduced Update Kalman Filter

In one dimension, the Kalman filter offers an attractive solution to the linear filtering and prediction problem. The extension of one dimensional Kalman filtering to two dimensions requires not only a suitable 2-D recursive model but also an enormous amount of data storage and transfer due to the high dimension of the resulting state vector. Hence a straightforward extension is of limited success, and thus it becomes desireable to consider computationally effective approximations. Here we review one such approximation, the 2-D reduced update Kalman filter as presented in [1].

To illustrate this approach, consider the scanning of a discrete 2-D field on an NXN regularly spaced lattice. Since the scanning operation does not qualitatively affect the results, we assume a raster scan.

We now consider a signal model which is Markovian and given by a non-symmetric half plane (NSHP)

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recursive model.[2]

$$s(m,n) = \sum_{\mathcal{R}_{\oplus+}} c_{kl} s(m-k,n-1) + w(m,n) \qquad (1)$$

where $w(m,n)$ is a white Gaussian noise field and $\mathcal{R}_{\oplus+}$ is an NSHP, i.e. $\{m>0,\ n>0\} \cup \{m<0,\ n<0\}$. We assume this model is (MxM)th order. The observation model is

$$r(m,n) = s(m,n) + v(m,n) \qquad (2)$$

where $v(m,n)$ is a white Gaussian source. Using the scanning operation we transform the 2-D problem into an equivalent 1-D problem. Define a state vector of $m(n+1)$ componants,

$$\underline{s}(m,n) = [s(m,n),\ s(m-1,\ n),\dots,s(1,n);\ s(n,n-1),\ ..s(1,n-1);\dots;s(n,\ n-m),\dots,s(m-m,\ n-m)]^T$$

then (1) and (2) can be put into the form,

$$\underline{s}(m,n) = \underline{C}\ \underline{s}(m-1,n) + \underline{w}(m,n), \qquad (3)$$

$$r(m,n) = \underline{H}\ \underline{s}(m,n) + v(m,n) \qquad (4)$$

Thus, we could immediately write down the Kalman equations[1] with the above interpretation of the s vector. The difficulty with these equations is the amount of computation and memory requirements associated with them. By limiting the update process to only those elements "near" the 'present' point, the computation can be greatly reduced. The resulting reduces update Kalman filter equations can be written in scalar form as given below. For details see [1]. In these equations, the superscript indicates the step in the filtering, while arguments represent the position of the data on the nxn grid.

State Prediction and update:

$$\hat{s}_b^{(m,n)}(m,n) = \sum_{\mathcal{R}_{\oplus+}} c_{k\ell}\ \hat{s}_a^{(m-1,n)}(m-k,n-\ell) \qquad (5)$$

$$\hat{s}_a^{(m,n)}(i,j) = \hat{s}_b^{(m,n)}(i,j) + K^{(m,n)}(m-i,n-j)\cdot[r(m,n)-\hat{s}^{(m,n)}(m,n)]$$
$$(i,j)\in\mathcal{R}_{\oplus+}^{(m,n)} \qquad (6)$$

Error Covariance and Gain:

$$R_b^{(m,n)}(m,n;k,1) = \sum_{o,p} c_{op} R_a^{(m-1,n)}(m-0,n-p;k,L)$$
$$(k,\ell)\in\mathcal{S}_{\oplus+}^{(m,n)} \qquad (7)$$

$$R_b^{(m,n)}(m,n;m,n) = \sum_{kl} c_{kl} R_b^{(m,n)}(m,n;m-k,n-1) + \sigma_w^2 \qquad (8)$$

where $\mathcal{S}_{\oplus+}$ is the support of the state vector $\underline{s}$.

$$R_a^{(m,n)}(i,j;k,1) = R_b^{(m,n)}(i,j;k,1) - K^{(m,n)}(m-i,\ n-j)\cdot R_b^{(m,n)}(m,n;k,1),(i,j)\in\mathcal{R}_{\oplus+}^{(m,n)};\ (k,\ell)\in\mathcal{S}_{\oplus+}^{(m,n)} \qquad (9)$$

$$K^{(m,n)}(i,j) = R_b^{(m,n)}(m,n;i,j)/[R_b^{(m,n)}(m,n;m,n)+\sigma_v^2],\ (i,j)\in\mathcal{R}_{\oplus+}^{(m,n)} \qquad (10)$$

Further reduction in computation can be obtained by computing (7) and (9) in a fixed size region, smaller than $\mathcal{S}_{\oplus+}^{(m,n)}$. Such a region will be referred to as $\mathcal{T}_{\oplus+}^{(m,n)}$ (see Figure 1).

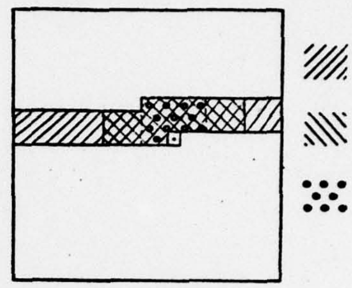

Fig. 1: Specifications of $\mathcal{S}, \mathcal{R}, \mathcal{T}$ used to Define Regions Updated.

Identification Algorithm

The implementation of the reduced update Kalman filter algorithm (5-10) requires the knowledge of the $c_{k\ell}$'s in (5). In the following, an algorithm is described for identification of these unknown parameters. Let us order all $c_{k\ell}\in\mathcal{R}_{\oplus+}$ into a column vector $\underline{c}$. Equaction (1) can then be written as:

$$s(m,n) = \underline{c}^T\underline{s}_1(m,n) + w(m,n), \qquad (11)$$

where $\underline{s}_1$ is the portion of the state vector $\underline{s}$ in the model's active memory, the pseudo-state vector[1].

Images are, in general, non-homogeneous and hence the elements of $\underline{c}$ are spatially variant. Ideally, then, a $\underline{c}$ vector should be found for each pixel (m,n). This, however, would involve a large amount of computation in both the identification and the reduced update Kalman filter algorithms. As a compromise of accuracy versus amount of computation, we assume regional homogeneity and find a estimate $\hat{\underline{c}}$ for each such region. The least-squares estimate of $\underline{c}$, for a KxK block of the NxN image, is then obtained by minimization of the following cost function:

$$J_K = \sum_{i,j}^{K,K}[s(i,j) - \hat{\underline{c}}^T\underline{s}_1(i,j)]^2 \qquad (12)$$

which yields (cf.[3,4]):

$$\hat{\underline{c}} = [\sum_{i,j}^{K,K}\underline{s}_1(i,j)\ \underline{s}_1^T(i,j)]^{-1}\cdot[\sum_{i,j}^{K,K}\underline{s}_1(i,j)s(i,j)] \qquad (13)$$

Effectively, we are implementing a fixed memory filter over the KxK block of an image.

The variance of the plant noise $w(K)$ associated with each $\hat{c}$ can be estimated using:

$$\hat{\sigma}_{w(K)}^2 = \frac{1}{K^2}\sum_{i,j}^{KK}[s(i,j)-\hat{\underline{c}}^T\underline{s}_1(i,j)]^2 \qquad (14)$$

The parameter estimate $\hat{c}$ and plant noise variance $\hat{\sigma}_w^2$ are then used by the reduced updated Kalman filter algorithm, as explained in the next section.

Combined Identification and Estimation

As previously mentioned, the image data field, in general, is not a stationary field and if represented by a constant set of parameters, a high plant noise can be expected. On the other hand, smaller blocks of the image data field are more likely to possess the stationarity property, and thus can be modelled with constant parameters more accurately.

Hence for simulation purposes, the 128 x 128 image data field was divided into 16 32 x 32 blocks. Then, the 1024 values of observations from each block were used to estimate the parameters of the state model parameter vector through (13), as well as to estimate the associated plant noise in each block (14).

These parameter values and associated plant noise were then passed on to the reduced update Kalman filter (5-10). From previous experiments, the required $\mathcal{T}_{\oplus+}$ and $\mathcal{R}_{\oplus+}$ regions were chosen as shown in Fig. 2. Boundary conditions for eqns. (7), (8) and (9) were assumed to be white Gaussian while those for eqn. (5) and (6) were assumed to be zero. Though the parameter and gain values were changed across the boundary of each block in the filter, no detectable edge effects were noted in the estimated image.

The estimated image was then compared with the noise-free data. Experimental results are given in the next section.

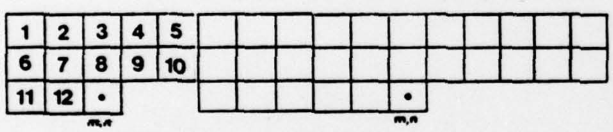

Figure 2: State and Covariance Update Regions Used for Experiments

Experimental Results

To examine the behavior of the identification algorithm, it was first applied to an image generated with a prior known parameters, to be explained in part A of this section. In part B, the application of the identification algorithm to a typical non-homogeneous image is discussed. This image, observed through equation (4), was then filtered by using the reduced update Kalman filter algorithm using the identified parameters.

A. Testing the Identification Algorithm

A homogeneous data field was generated with known parameters and plant noise variance, so as to satisfy (11) for all (m,n). The results of applying the identification algorithm are given in Table 1. As evident from these results, all estimates improve with an increase in block size, validating the consistancy of the identification procedure. The test data field was subsequently immersed in noise and filtered with the 2-D reduced update filter. The computed and measured mean square error were within 10% for the SNR = 3db test case.

B. Processing of Noisy Pictures

The noise-free image, as shown in Figure 3, is a 128 x 128 image data field. Additive measurement noise w(n) was simulated using a Gaussian white noise generating subroutine. For simulation purposes, a 3 db signal to noise ratio was used, with signal variance equal to 2884. This noisy-image was processed as described above. The process noise variance was experimentially optimized to produce the best MSE improvement.

Figure 4 shows the 3db noisy image. The estimated image, using a stationary second order $\oplus+$ NSHP model[2] is shown in Figure 5. The MSE was 164 equivalent to a 9.44 db improvement. Figure 6 shows the estimated image, using block processing as described in section 4. The MSE was 134 equivalent to a 10.32 db improvement. This represents about a 20% reduction in MSE. Subjectively the noise level seems greatly reduced, however it appears that some distortion has been introduced by the filtering in some blocks. To investigate this matter, the noise free image and the white noise field were seperately filtered with the results shown in Fig. 7a and 7b, respectively. Figures 7 show that the distortion is composed of colored noise that has been shaped by the recursive estimator to have a 'local' spectrum similar to that of the noise free image. Such a noise is known to be of increased objectionality over an equivalent amount of isotropic noise.[5] This effect is a direct result of the filter's being 'tuned' to the 'local' spectrum of the signal. Thus, although subjectively somewhat heightened by the block-wise constant model, this effect is felt to be fundamental to spatially varying filters. A goal of future work will be to ameliorate this effect. Possible approaches include reducing the block size. We also note this effect is most pronounced in flat portions of the image.

These preliminary results are, in a sense, upper bounds on what can be achieved in practice. This is because the model parameters were calculated from the noise free image. For the constant parameter model, the parameters could just as well be estimated from the noisey image (given the power of the white observation noise). However for the block-wise constant parameter model with small block sizes, use of the noisy data would be expected to degrade the parameter estimates, especially so a low SNR's. A possible solution to this problem is the use of prototype images.[6] The motivation for doing this initial work involving noise free parameter estimation was to try, in so far as possible, to separate the effects of spatially varying filtering from the effects of statistical errors in the parameter estimates. Future work will account for the bias introduced into the parameter estimates by the observation noise using the method of [7].

Conclusions

The above results show that spatially variant recursive estimation can significantly improve image quality compared to spatially invariant or constant parameter processing. Conversely, spatially variant processing tends to correlate input noise in a manner similar to the 'local' correlation of the signal, thus increasing subjective objectionality in flat portions of the image.

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Fig. 3 Noise Free Image

Fig. 4 Noisy Image 3-db

Fig. 5 Estimate Using Constant Model Param.

Fig. 6 Estimate Obtained Using Block-wise Constant Model Parameters

Fig. 7a Filtered Noise Free Image

Fig. 7b. Filtered White Noise

Model Parameters

K	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	$\hat{\sigma}_w^2$	ε^2 *
True Values	.138	-.048	-.096	.03	-.04	.033	-.61	.839	-.048	.046	-.231	.968	216.29	
4	-.04	.57	.35	-.27	.29	.20	-.62	1.46	-1.49	1.16	-.40	1.102	32.96	1.43
8	.15	-.29	.42	-.51	.27	.17	-.64	.50	.48	-.28	-.35	1.11	159.7	1.57
16	.13	-.08	-.00	-.07	-.008	.07	-.70	.90	.008	-.02	-.27	1.024	203.99	.89
32	.16	-.09	-.05	-.01	-.02	-.06	-.54	.84	-.01	.018	-.15	.92	207.17	.449
64	.14	-.07	-.08	.029	-.05	-.002	-.56	.85	-.03	.04	-.19	.927	217.91	.23
128	.1375	-.056	-.089	.023	-.04	.018	-.59	.841	-.048	.048	-.214	.956	216.12	.118

Table 1. Parameter Estimates vs Block-size

$$* \ \varepsilon^2 \triangleq \sum_{i=1}^{12} (\hat{c}_i - c_i)^2$$

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This paper presents initial results on spatially variant recursive estimation of images. Parameters of block-wise constant recursive model are identified on noise free data. The models are then used to design reduced update Kalman filters which are applied to noisy data. The results are presented and discussed.

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